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Report on the scholarship period

During my 4 weeks long visit to the University of Sopron, I have worked with professor Laszlo Szalay. As it was planned, we firstly discussed the various problems concerning the generalizations of Diophantine sets. Diophantine sets are the sets consisting of m positive integers, such that product of any two of them increased by 1 is a perfect square. The question of interest is how large those sets can be. Recently it was proven (by He, Togbe and Ziegler) that we cannot have 5 element in such set. However, there are various generalizations of the original problem. One is to look for such sets in different rings, the others are looking for different perfect powers instead of squares or adding different non-zero integer n instead of 1. In all those cases it is also of interest to see how large those sets can be.

After discussing what would be the most appropriate version to work on and make a progress, we have decided to firstly solve one problem of Triangular Diophantine m-tuples. More precisely, we have proved that there does not exist a set $\{1, 2, c, d\}$ of four positive integers such that a product of any two of its elements increased by 1 is triangular number. Triangular number are those of the form $x(x+1)/2$ where x is a positive integer. In the proof, we first found all possible values of c that will extend our pair $\{1, 2\}$. After that, we have transformed the problem of extending such triple to a quadruple to solving the system of simultaneous Pell-like equations. It is a standard problem that furthermore leads to finding intersections of binary recurrence sequences, which is then solved combining congruence method and Baker's theory on linear forms in logarithms of algebraic numbers. That work is finished and the joint paper is already submitted to a journal.

Except that, we have also discussed other problems which we will continue to work on in the next period, and it will certainly result in more joint papers in future.

Also, during the last week of my stay, I have given a seminar talk on my recent research at University of Sopron, for which I hope will also interest some attendants to this field of research.

Alan Filipin